

# Technological Progress and Capital Obsolescence: Implications for Productivity Growth\*

Adeliada Mehmetaj<sup>†</sup>

Drexel University

November 22, 2018

## Abstract

This paper examines how endogenous capital depreciation due to the replacement of obsolete capital-embodied technologies can lead to mismeasurement of productivity growth. In an environment where technology is capital-augmenting, the optimal decision to retire old, less-productive capital increases in the rate of economic growth. Failure to allow depreciation to vary with the state of the economy leads to mismeasurement of the capital stock and implies that Total Factor Productivity (TFP) growth is a biased metric of true productivity growth. I show that approximately 15% of the productivity slowdown in late-1970s to mid-1990s, is due to a higher bias following a rapid acceleration in the rate of capital-embodied technological innovation. In turn, the true slowdown in productivity since mid-2000s is underestimated by around 17%.

*Keywords:* Endogenous Depreciation, Capital-Embodied Technology, TFP Growth, Productivity Slowdown.

*JEL Codes:* E01, E22, E32.

---

\*I sincerely thank my dissertation advisor André Kurmann, my committee members Maria Pia Olivero, Christopher Laincz, Marco Airaudo, seminar participants at Drexel University and participants at the EEA conference for their useful suggestions. All remaining errors are my own.

<sup>†</sup>*Address:* Drexel University, LeBow College of Business, School of Economics, 3220 Market Street, Philadelphia, PA 19104; *Email:* adeliada.mehmetaj@drexel.edu; *Website:* adeliadamehmetaj.com

# 1 Introduction

Capital is an important factor of production and as such the assumptions we make about capital accumulation have important implications for measuring productivity growth. The literature generally assumes that new and old capital can be combined seamlessly and capital depreciation is a constant. These are both restrictive assumptions since newer capital embodies better/more innovative technologies, which in turn may render older capital obsolete. For example, once faster computers became available firms started switching older versions for them. There is a substitutability between new and old capital vintages that the standard capital accumulation equation does not incorporate. Failing to account for this substitutability results in an upward-biased capital stock measure. Overstating capital stock, renders TFP growth rate computed from standard growth accounting to be a biased estimate of true productivity growth.

To understand and quantify the bias, I revisit this problem in a dynamic stochastic general equilibrium (DSGE) model with putty-clay investment, in line with Gilchrist and Williams (2000).<sup>1</sup> One unit of capital is represented by a machine. Technological progress is capital-augmenting and the firm decides to invest in new machines before a machine-specific idiosyncratic shock is realized. Then, as machines of the same vintage are subject to different idiosyncratic shocks, the putty-clay model incorporates the desired property that not all capital of a given vintage is retired. The putty-clay model is also capable of replicating important business cycle data facts and it incorporates capital obsolescence in a tractable fashion. If the realized shock is high enough such that a machine's productivity exceeds the variable cost of production (i.e. wage rate), the machine is used to produce. Otherwise, the firm optimally decides to retire it from production. Endogenous depreciation in this model results from the assumption that *ex-post* capital-labor ratios are fixed and the firm is unable to adjust labor to meet the marginal productivities of each vintage of capital. Two important results of the

---

<sup>1</sup>Investment is putty-clay in that, once the level of technology is embedded in a unit of capital, it cannot be changed or updated to incorporate better capital-embodied technologies as they become available.

model for the purpose of measuring capital and productivity are: aggregate output is no longer a Cobb-Douglas production function; and the decision to retire obsolete capital is endogenous and depends on the wage rate and thus on the state of economic growth. These results contrast with the common practice of computing TFP growth both in models and by the BEA.

The BEA measures capital stock using depreciation and investment data from the National Income and Product Accounts (NIPAs). In the NIPAs investment series is adjusted to allow for capital-embodied technological progress in line with Gordon's (1990) hedonic methods and depreciation adjusted for obsolescence is computed as a geometric constant in line with Hulten and Wykoff (1981). This adjustment is done in an ad-hoc way and does not capture the idea of capital obsolescence as a function of economic growth. When the economy moves to a higher state, obsolescence from the NIPAs underestimates the true depreciation. Therefore, as the growth rate of capital-embedded technological progress increases, capital stock computed by the BEA is biased upward which in turn results in a negatively biased measure of TFP growth.

When I compute TFP growth rate using a standard growth accounting equation on the Balanced Growth Path (BGP) of the putty-clay model, I find that both the departure from Cobb-Douglas aggregation and endogenous depreciation contribute to a bias that is a function of the state of the economy. This bias is affected positively by the lack of aggregation in a Cobb-Douglas and negatively by the endogenous depreciation. For realistic growth rates of capital-embodied technology, the negative bias from endogenous depreciation dominates the positive bias due to the absence of aggregation.

The paper focuses on two particular episodes of productivity slowdown: late-1970 to mid-1990s and mid-2000s onward. To quantify the bias, I consider the time series of the relative price of total investment from the FRED dataset, which is available quarterly from 1947 until 2016. According to the model, the relative price of investment is an inverse measure of capital-embodied technological progress. Then, I use this series to compute the capital-embodied

technology and its growth rates. I find an acceleration in the rate of technological innovation in late-1970s to mid-1990s and a deceleration in mid-2000s. I compute the average annual productivity bias to find that it increased from 0.38% to 0.52% in late-1970s and it decreased from 0.74% to 0.54% in mid-2000s. As also argued previously, the bias is positively related to the state of the economy. Then I use the TFP growth series computed by Fernald (2014), using capital stock constructed from the NIPAs, to look at the implications for productivity growth. I find that 15% of the slowdown in productivity growth in the early-1980s is explained by an increase in the bias due to the fast acceleration in capital-embodied technological progress of the late-1970s. This in part explains Robert Solow's productivity paradox: "You can see the computer age everywhere but in the productivity statistics". Most importantly, my results show that the true slowdown in recent productivity is more severe than what Fernald's (2014) TFP growth measure indicates. Accounting for the decline in technological innovation and along with it productivity bias in mid-2000s, I find that the change in TFP growth rate computed by Fernald (2014) understates the true slowdown by 17%.

Relative to the literature, this paper is most closely related to Mukoyama (2008), who argues that the TFP slowdown in Greenwood et al. (1997) can be largely explained by the bias due to endogenous depreciation. He finds that a 2.6% underestimation of endogenous depreciation, results in around 0.9% underestimation in productivity growth. The main departure from Mukoyama (2008) is that I explore the specific channel through which the bias in endogenous depreciation propagates to mismeasurements in TFP growth. Specifically, I show that it is crucial to understand how the co-movement between the growth rate of capital-embodied technological progress and the TFP growth can be used to compute the true slowdown in productivity. Additionally, I identify two sources of bias: Cobb-Douglas aggregation and endogenous depreciation. I find that they have opposite effects on the rate of TFP growth. Most importantly, I extend the sample and use these new insights to quantify the implications of the

bias for the recent productivity slowdown.

To model the substitutability between new and old capital, I follow Gilchrist and Williams (2000) closely. Gilchrist and Williams (2000) use the putty-clay investment model to show that capital-embodied technologies can be an important source of business cycle fluctuations. The model replicates important stylized facts such as the co-movement between output and labor and positive autocorrelation of output. Campbell (1998) uses a similar approach to study the entry and exit of firms during the business cycle and finds that entry is procyclical while exit is countercyclical. Another similar approach to modeling endogenous depreciation of capital is introduced by Cooley et. al. (1997). Within a vintage capital model with capital-embodied technology, economic depreciation is defined explicitly as the endogenous decision to replace old capital with new more productive capital. Gilchrist and Williams (2000) is preferred because of the property of not having to keep track of vintages in equilibrium. Greenwood et al. (1988) offer another explanation for endogenous depreciation. Within their model, endogenous depreciation arises from the firm's decision to increase the utilization of more productive capital as it becomes available. Depreciation varies with the state of the economy, but it is due to an increase in wear and tear rather than more obsolescence.

Quantitatively, this paper is related to Greenwood et al. (1997) who find that productivity declined rapidly following a rapid increase in capital quality; and Fernald (2014) who shows that there has been an ongoing slowdown in productivity growth starting since before the Great Recession. In Greenwood et al. (1997) it is assumed that depreciation is a constant due to wear and tear. As a result, they compute a negative growth rate of TFP following a rapid increase in capital-embodied technology. In Fernald (2014), obsolescence measures are adopted from the NIPAs. However as explained previously the adjustment for obsolescence in the NIPAs is done in an ad-hoc way and obsolescence is a constant. Therefore the TFP growth series computed by Fernald (2014) is still positive, but its trend has become less steep. We add to their stories in

that: allowing endogenous depreciation to be a function of the state of the economy, explains around 15% of the slowdown computed by Greenwood et al. (1997) and we argue that the recent slowdown is in fact even more severe than what is shown by Fernald (2014).

## 2 Model

This section introduces a dynamic stochastic general equilibrium model with putty-clay production technology in line with Gilchrist and Williams (2000). At any period  $t$ , project managers observe the economy-wide level of capital-augmenting technological progress  $\theta_t$  and decide to invest  $k_t$  units of capital in a new machine  $X_t$ . After  $k_t$  is chosen, a machine-specific idiosyncratic shock  $\mu_t$  is revealed, such that any machine  $X_t = \mu_t \theta_t k_t^\alpha$ , for  $\alpha$  denoting the usual capital elasticity of output. The capital-augmenting technology  $\theta$  has a mean gross growth rate of  $(1 + g_\theta)^{1-\alpha}$  and the idiosyncratic shock  $\mu$  is lognormally distributed (i.e.  $\log(\mu) \sim N(-\frac{1}{2}\sigma^2, \sigma^2)$ ), with a probability density function (pdf)  $f_\mu(\cdot)$ . Under the assumption of constant returns to scale, each machine requires one unit of labor at full capacity to produce. The only cost of production is the wage rate  $W$ , which cannot be adjusted to meet the marginal productivity of each machine. This introduces a kink in the production function, where at any period, any machine whose marginal productivity is lower than the wage rate will be separated. A new machine requires one period of initial installation and it can be used to produce at any future period thereafter. Machines of a certain efficiency  $X$  evolve as follows:

$$H_{t+1}(X) = (1 - \delta)H_t(X) + f_X(X; \bar{X}_t)Q_t, \quad (1)$$

where

$$f_X(X; \bar{X}_t) = Pr(X_t = X; \bar{X}_t) = \frac{1}{\bar{X}_t} f_\mu\left(\frac{X}{\bar{X}_t}\right)$$

is the pdf of  $X_t$ ,  $\bar{X}_t = E(X_t)$ , and  $Q_t$  is the number of machines put in place at time  $t$  defined as  $Q_t = \frac{I_t}{k_t}$ , for  $I_t$  being aggregate investment. The total stock of machines at any period, is the sum of all old machines minus the share that depreciated due to wear and tear at rate  $\delta$ , and the new machines that are put in place.

Then, aggregate output at any period  $t$  is defined as

$$Y_t = \int_{X \geq W_t} A_t X H_t(X) d(X), \quad (2)$$

where  $A_t$  stands for neutral technology. Substituting for  $H_t(X)$  and integrating forward, results into the following closed form solution for output

$$Y_t = \sum_{j=1}^{\infty} A_t (1 - \delta)^{j-1} [1 - \Phi(z_t^{t-j} - \sigma)] \bar{X}_{t-j} Q_{t-j}, \quad (3)$$

for  $\Phi(\cdot)$  representing the cumulative distribution function (cdf) of the standard normal and  $z_t^{t-j} = \frac{1}{\sigma} (\log W_t - \log \bar{X}_{t-j} + \frac{1}{2} \sigma^2)$  being the lognormally-adjusted production threshold. The resulting aggregate output is the product of current neutral technology  $A_t$  and the sum of the shares of past machines whose productivity exceeds the wage rate. Clearly, the assumption of a Cobb-Douglas aggregate production function no longer holds in this model.

## 2.1 The Investment Decision

The level of capital  $k_t$  embedded in each new machine  $X_t$  is chosen such that the lifetime profit from the new machine is maximized as follows:

$$\max_{k_t} \left\{ -k_t + E_t \sum_{j=1}^{\infty} \pi_{t+j}^t \right\}. \quad (4)$$

The price of a new machine is normalized to one, and  $\pi_{t+j}^t$  is the profit of a machine  $X_t$  at any future period  $t + j$  defined as:

$$\pi_{t+j}^t = (1 - \delta)^{j-1} \int_{X_t > W_{t+j}} R_{t,t+j} (A_{t+j} X_t - W_{t+j}) dF(X_t; \bar{X}_t), \quad (5)$$

for  $R_{t,t+j}$  being the discount rate. Solving forward, results in

$$\pi_{t+j}^t = (1 - \delta)^{j-1} R_{t,t+j} \{ (1 - \Phi(z_{t+j}^t - \sigma)) A_{t+j} \bar{X}_t - (1 - \Phi(z_{t+j}^t)) W_{t+j} \}, \quad (6)$$

where with probability  $(1 - \Phi(z_{t+j}^t - \sigma))$  the machine produced at time  $t$  will survive separation at time  $t + j$ , in which case the firm's profit will be the difference between the machine's productivity  $X_t$  and the current wage rate  $W_{t+j}$ . In case of separation, the machines have a scrap value of zero. The first order condition necessary for an interior solution of  $k_t$  requires that the cost of investing in a new machine equals the sum of all its future profits:

$$k_t = \alpha E_t \sum_{j=1}^{\infty} (1 - \delta)^{j-1} R_{t,t+j} (1 - \Phi(z_{t+j}^t - \sigma)) A_{t+j} \bar{X}_t. \quad (7)$$

New machines will be put in place as long as the following zero-profit condition is satisfied:

$$k_t = E_t \sum_{j=1}^{\infty} (1 - \delta)^{j-1} R_{t,t+j} \{ (1 - \Phi(z_{t+j}^t - \sigma)) A_{t+j} \bar{X}_t - (1 - \Phi(z_{t+j}^t)) w_{t+j} \}. \quad (8)$$

The total capital stock used in production at any period  $t$ , which here will be referred to as the "effective capital stock", equals the total labor<sup>2</sup>. Denoting the effective capital stock as  $K_t^e$ , we have  $K_t^e = L_t$ , for

$$L_t = \int_{X \geq W_t} H_t(X) d(X). \quad (9)$$

Solving forward we express total labor at any period as the sum of the total shares of machines

---

<sup>2</sup>Recall that one machine requires exactly one unit of labor to produce.



which survive separation:

$$L_t = \sum_{j=1}^{\infty} (1 - \delta)^{j-1} [1 - \Phi(z_t^{t-j})] Q_{t-j}. \quad (10)$$

Then, endogenous depreciation at period  $t$  which here is denoted as  $d_t$  equals the total share of machines that are left unused

$$d_t = \int_{X \leq W_t} H_t(X) d(X). \quad (11)$$

## 2.2 Households

The household's role in this economy is standard. The representative household chooses consumption  $C_t$ , labor  $L_t$  and a risk-free asset to smooth consumption  $B_{t+1}$ , to optimize its log-utility preferences

$$V(B_t) = \max_{C_t, L_t, B_{t+1}} \{ \ln(C_t) + \xi \ln(1 - L_t) + \beta E_t V(B_{t+1}) \}, \quad (12)$$

subject to the budget constraint

$$C_t + B_{t+1} = w_t L_t + (1 + r_t) B_t + D_t, \quad (13)$$

where bonds earn interest  $r_t$ ,  $\beta \in (0, 1)$ ,  $\xi > 0$  and the dividends received as owners of firms

$$D_t = Y_t - w_t L_t - I_t.$$

The first order conditions are standard as follows:

$$w_t = \xi \frac{C_t}{1 - L_t}, \text{ and} \quad (14)$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} (1 + r_{t+1}). \quad (15)$$

In a closed economy with identical agents, market clearing implies that bonds are zero in net supply:  $B_{t+1} = B_t = 0$ . Substituting for zero bonds and the dividend equation in the household's budget, we retrieve the budget constraint of the economy  $C_t = Y_t - I_t$ . Additionally, in equilibrium the discount rate  $R_{t,t+1} = \beta E_t \frac{C_t}{C_{t+1}}$ , which implies that

$$R_{t,t+1} = (1 + r_{t+1})^{-1}. \quad (16)$$

### 3 Results

This section considers the putty-clay model on the BGP to quantify the contribution of capital mismeasurement to productivity slowdown. Let us start by assuming that the growth rate of capital-embodied technology  $g_\theta$  is the only source of growth along the BGP. Then, from the budget constraint  $C = Y - I$  output, consumption and investment grow at the same gross rate, call it  $(1 + g)$ . For given vintage  $j$ , the threshold on the BGP is

$$z(j) = \frac{1}{\sigma} [\ln(w) - \ln((1 + g)^{-j} \bar{X}) + \frac{1}{2} \sigma^2]. \quad (17)$$

Using the BGP output equation and the first order condition respectively

$$y = q \bar{X} \sum_{j=1}^M (1 - \delta)^{j-1} [1 - \Phi(z(j) - \sigma)] (1 + g)^{-j}, \text{ and} \quad (18)$$

$$k = \alpha \bar{X} \sum_{j=1}^M [\beta(1 - \delta)]^{j-1} [1 - \Phi(z(j) - \sigma)] (1 + g)^{-j}, \quad (19)$$

I find that both the average machine  $\bar{X}$  and its capital installed  $k$  also grow at a rate  $(1 + g)$ .

Then the definition of a machine  $\bar{X} = \theta k^\alpha$  results in the following relation between  $g$  and  $g_\theta$

$$g = g_\theta^{\frac{1}{1-\alpha}}. \quad (20)$$

In line with the long run growth facts, labor is constant at 0.2, hence its growth rate is zero and for simplicity I also assume that the growth rate of neutral technology  $g_A = 0$ .

### 3.1 Understanding the Bias

To understand and quantify the bias coming from the assumption of constant depreciation, here I compute the TFP growth rate, call it  $g^{TFP}$ , on the BGP using the standard growth accounting technique:

$$g^{TFP} = g^y - \alpha g^K - (1 - \alpha)g^L. \quad (21)$$

There are two implicit assumptions that allow us to write the growth accounting problem this way: output is aggregated as a Cobb-Douglas and total capital results from a standard capital-accumulation equation with capital-embodied technological progress and a constant depreciation rate  $\delta$  as follows:

$$K_{t+1} = (1 - \delta)K_t + \theta_t I_t. \quad (22)$$

This measure differs from the previously defined effective capital, which is computed under the assumption of endogenous depreciation and cannot be generalized with a standard accumulation equation. In line with Gilchrist and Williams (2000), here I assume that  $\delta = 0.021$  per quarter. Then, the implied growth rate of capital along the BGP is

$$g^K = g_\theta + g_I. \quad (23)$$

The growth rates of output, investment and labor are pinned down from the putty-clay model (i.e.  $g_y = g_I = g$  and  $g_L = 0$ ). Assuming a relevant interval for the rate of capital-embodied technology  $g_\theta \in [0, 0.05]$ , I compute a corresponding rate  $g^{TFP}$  for each  $g_\theta$  on the BGP. I find that there is a discrepancy between  $g^{TFP}$  and  $g_A$ . This represents the bias in computed TFP growth. As shown in Figure 1, this bias increases in the rate of technological progress

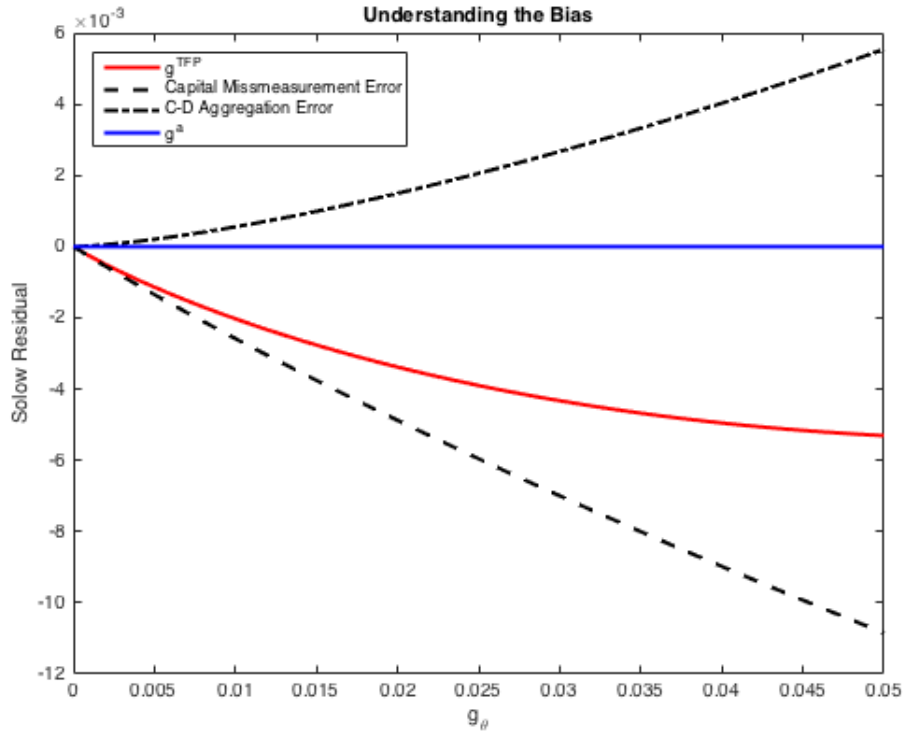


Figure 1: This graph shows the growth rate in TFP at each rate of capital-augmenting technological progress along the BGP.

$g_\theta$ , which means that the error in computing the growth rate of neutral technology using a standard growth accounting approach is higher at higher rates of technological innovation. There are two sources of bias, one being the assumption of the Cobb-Douglas aggregation which biases the results upwards and the other being endogenous depreciation, which when violated makes the bias even more negative. If I were to extend the graph to include higher rates of technological progress (beyond 0.05), the bias due to the Cobb-Douglas aggregation assumption would dominate resulting in the computed growth rate of TFP to be closer to that of neutral productivity. Then, using standard growth accounting to compute productivity growth would not be too far from the truth.

A different choice for the constant depreciation rate  $\delta$ , will change the level of the bias without affecting the trend. For example, choosing a higher constant value for  $\delta$  will reduce the level of the bias, but the discrepancy between the new TFP growth and the rate of neutral technology will still be increasing as the growth rate of capital-augmenting technology increases.

### 3.2 Quantifying the Bias and Productivity Slowdown

I use the relative price series for total investment from the FRED database to compute the growth rates in capital-embodied technological progress as the growth rate of its inverse. The data is available quarterly from 1947 until 2016. Call the relative price of investment at time  $t$   $p_{I,t}$ , then

$$\theta_t = \frac{1}{p_{I,t}}, \text{ and} \quad (24)$$

$$g_{\theta,t+1} = \frac{\theta_{t+1}}{\theta_t} - 1. \quad (25)$$

The relative price of investment and the capital-embodied technology series are both shown in Figure 2. In line with the rest of the literature, the graphs indicate an acceleration in innovation in the early-1980s to mid-1990s (see Cummins and Violante (2002), Greenwood et al. (1997) etc) and a deceleration after mid-2000s. To quantify the rate of technological innovation in these sub-periods and its implications for productivity growth, I separate the sample into 4 sub-samples: 1947 – 1979, 1979 – 1995, 1995 – 2005, 2005 – 2016. I compute the average annual technology growth rate  $\bar{g}_\theta$  for each sub-sample and the corresponding bias, which I show in Table 1.

Table 1: Capital-embedded Technology Growth and the Implied Bias

<b>Time Period</b>	$\bar{g}_\theta(\%)$	<b>Bias (%)</b>
<b>1947-1979</b>	1.61	-0.38
<b>1979-1995</b>	2.36	-0.52
<b>1995- 2005</b>	3.54	-0.74
<b>2005-2016</b>	2.46	-0.54

Based on these computations, the average annual growth rate in technology increased from 1.61% to 2.36% after 1979. The absolute bias also became more pronounced; from 0.38% to 0.52%. The opposite trend in technological innovation is computed after 2005, where both

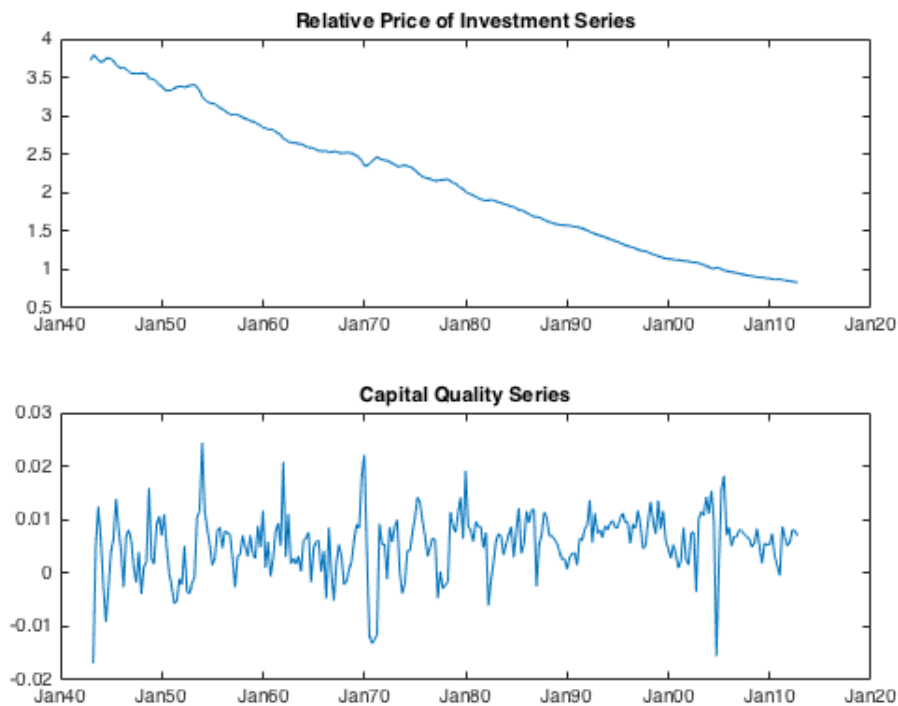


Figure 2: This figure shows the time series of the relative price of total investment (top) and the corresponding time series of capital-augmenting technology (bottom).

average annual technology and the bias decreased from 3.54% to 2.46% and from 0.74% to 0.54% respectively. Using the price series of equipment instead of total investment results in higher, more significant differences.

Next, I use the TFP growth series computed by Fernald (2014) to quantify the implications for productivity growth. Fernald (2014) uses investment series adjusted for capital-embodied technological progress and obsolescence data from the NIPAs. As explained earlier in the paper, the NIPAs obsolescence measure is computed in a ad-hoc way and is constant for given types of equipment and structures. Therefore we should expect to see a bias in this TFP growth series, since obsolescence measures used to compute capital stock do not vary with the state of the economy.

### 3.2.1 The Growth Slowdown of early-1980s to mid-1990s

“You can see the computer age everywhere but in the productivity statistics”, Solow (1987). Robert Solow’s famous Productivity Paradox hints that there is a discrepancy in the way that technological innovation typically affects measures of productivity in our models. The average TFP growth computed by Fernald (2014), indicates a slowdown in the growth rate of productivity in the late-1970s from 1.61% to 0.58% following the rapid acceleration in technology. In this paper, I show that due to the increase in bias parallel to the acceleration in technology, the true decline in productivity is not as severe.

Denoting the average TFP growth rate as  $g^{TFP}$ , I calculate the true average growth rate  $g^{True}$  after adjusting for the bias as follows:

$$g^{True} = g^{TFP} - Bias. \quad (26)$$

The results show that the actual growth rate of neutral technology decreased from 1.99% to 1.10%. Referring to Table 3, the growth rate as computed by Fernald (2014) during this period decreased by  $-1.09\%$ , while the true growth rate that I compute only fell by  $-0.95\%$ . These results indicate that the Slowdown following the period of high innovation in the late-1970s was not as severe. Specifically, adjusting for the bias makes the slowdown 15% less severe.

The variables shown in Table 3, namely  $\Delta g^{TFP}$  and  $\Delta g^{true}$  denote the change in a given growth rate, either  $g^{TFP}$  or  $g^{True}$ , computed as follows:

$$\Delta g_{\tau}^{TFP} = g_{\tau}^{TFP} - g_{\tau-1}^{TFP}, \text{ and} \quad (27)$$

$$\Delta g_{\tau}^{True} = g_{\tau}^{True} - g_{\tau-1}^{True}, \quad (28)$$

for  $\tau$  being a time interval such as 1995 – 2005.

### 3.2.2 The Growth Slowdown from the mid-2000s onward

Fernald’s (2014) main argument is that there is a productivity slowdown which started since before the Great Recession and still persists. Specifically, the growth rate in TFP declined in mid- 2000s from 1.75% to 0.56%. As shown previously, this decline follows a decline in technological innovation, hence the bias coming from capital mismeasurement also declined. Referring to the results presented in Table 2, the true drop in productivity is even larger; productivity growth decreased from 2.49% to 1.10%. In Table 3, I compute the slowdown implied by Fernald (2014) to be  $-1.19\%$  while the true slowdown should be 17% larger, equal to  $-1.38\%$ . Therefore, due to a decline in technological innovation, the TFP series computed by Fernald (2014) underestimates the true drop in productivity growth in the recent years by 17%.

Table 2: TFP and Productivity Growth Rates

<b>Time Period</b>	$g^{TFP}(\%)$	<b>Bias (%)</b>	$g^{True}(\%)$
<b>1947-1979</b>	1.61	-0.38	1.99
<b>1979-1995</b>	0.58	-0.52	1.10
<b>1995- 2005</b>	1.75	-0.74	2.49
<b>2005-2016</b>	0.56	-0.54	1.10

Table 3: Changes in TFP and Productivity Growth Rates

<b>Time Period</b>	$\Delta g^{TFP}(\%)$	$\Delta g^{True}(\%)$
<b>1947-1979 and 1979-1995</b>	-1.10	-0.95
<b>1995-2005 and 2005-2016</b>	-1.19	-1.38

## 4 Extensions

So far I assume zero growth rate of neutral productivity. A possible extension of the model, is to allow for the growth rate of neutral productivity to vary and be pinned down by the model.



This will add another source of growth on the BGP of the putty-clay model. I believe that this extension will alter the productivity bias, such that on the BGP, for given growth rate of capital-embodied technology, the true productivity growth rate as defined earlier in the paper, will be exactly equal to the growth rate of neutral technology in the putty-clay model.

Another possible extension, is to allow for reallocation of used capital within the putty-clay model. For now I assume that all capital whose productivity is below the marginal cost of production becomes obsolete. This is a restrictive assumption which can potentially impose a larger downward bias on total capital. Allowing for used capital reallocation in the putty-clay model with endogenous depreciation, would adjust endogenous depreciation downward, implying a smaller productivity bias. I believe that the trend of the productivity bias will be unaffected by this extension.

## 5 Concluding Remarks

This paper studies the growth accounting implications of relaxing the assumption of constant depreciation in a putty-clay model with capital-embodied technological progress and endogenous capital depreciation. At the BGP endogenous depreciation increases in the rate of capital-embodied technological progress. Failure to account for endogenous depreciation overestimates capital which introduces a negative bias in computed TFP growth. This bias is larger for higher growth rates of capital-embodied technology. Using the relative price of total investment, I find that the average growth rate of capital-embodied technology increased in the late-1970s and decreased in mid-2000s. Using this result to compute the corresponding average bias in TFP growth, I show that the bias also increased in late-1970s and decreased in mid-2000s. Adjusting the TFP growth series for this bias, results in the conclusion that the severity of the TFP slowdown was overestimated during the late-1970s by 15% and underestimated for the Great Recession period by 17%.

## References

- [1] Campbell, Jeffrey R. “*Entry, Exit, Embodied Technology, and Business Cycles*”, Review of Economic Dynamics, 1998.
- [2] Cooley, Thomas F., Jeremy Greenwood and Mehmet Yorukoglu “*The Replacement Problem*”, Journal of Monetary Economics, 1997.
- [3] Cummins, Jason G., and Giovanni Luca Violante “*Investment-Specific Technical Change in the US (1947-2000): Measurement and Macroeconomic Consequences*”, Review of Economic Dynamics, 2002.
- [4] Fernald, John G. “*Productivity and Potential Output Before, During, and After the Great Recession*”, Federal Reserve Bank of San Francisco, Working Paper Series, 2014.
- [5] Fraumeni, Barbara M. “*The Measurement of Depreciation in the U.S. National Income and Product Accounts*”, Survey of Current Business, 1997.
- [6] Gilchrist, Simon, and John C. Williams. “*Putty Clay and Investment: A Business Cycle Analysis*”, Journal of Political Economy, 2000.
- [7] Gordon, Robert J. “*The Measurement of Durable Goods Prices*”, University of Chicago Press, 1990.
- [8] Greenwood, Jeremy, Zvi Hercowitz and Gregory W. Huffman. “*Investment, Capacity Utilization, and the Real Business Cycle*”, The American Economic Review, 1988.
- [9] Greenwood, Jeremy, Zvi Hercowitz and Per Krusell. “*Long run implications of Investment Specific Technological Change*”, The American Economic Review, 1997.
- [10] Hobijn, Bart. “*Embodiment in U.S. Manufacturing*”, Working Paper, 2001.
- [11] Hulten, Charles R., and Frank C. Wykoff “*The estimation of economic depreciation using vintage asset prices: An application of the Box – Cox power transformation*”, Journal of Econometrics, 1981.
- [12] Katz, Arnold and Shelby W. Herman “*Improved Estimates of Fixed Reproducible Tangible Wealth, 1929 – 95*”, Survey of Current Business, 1997.
- [13] Mukoyama, Toshihiko “*Endogenous depreciation, mismeasurement of aggregate capital, and the productivity slowdown*”, Journal of Macroeconomics, (2008).
- [14] Sakellaris, Plutarchos and Daniel Wilson. “*Quantifying Embodied Technological Change*”, Review of Economic Dynamics, 2004.
- [15] Whelan, Karl. “*A guide to U.S. Chain Aggregated NIPA Data*”, Review of Income and Wealth, 2000.