Capital Search and the Business Cycle

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Abstract

This paper presents a Real Business Cycle model with separate markets for new and used capital, where the reallocation of used capital is subject to search frictions. Households supply new capital to firms and used liquid capital comes from the separation of existing firm-capital relationships. In the general model, used capital separation is an endogenous choice of the firm. We compare this to the case where we fix the separation rate exogenously. Both cases, with and without endogenous separation show significant output amplification as compared to the baseline RBC model. The exogenous separation specification performs well in replicating the procyclical used capital reallocation that we see in the data. Reallocation becomes countercyclical when separation decision is endogenous, with the benefits to reallocation being counterfactually procyclical.

1 Introduction

Most modern DSGE models assume that capital reallocation is frictionless. Frictionless reallocation guarantees the existence of a Walrasian equilibrium, where price adjusts such that total supply equals total demand. Hence, these models rule out the existence of idle (unmatched) capital, which we see in the data. Such models also fail to explain the Eisfeldt and Rampini (2006) puzzle of capital reallocation. Eisfeldt and Rampini (2006) find that capital reallocation is procyclical while the benefits to reallocation are countercyclical. Countercyclical benefits to reallocation should imply countercyclical reallocation, yet the data shows the opposite. Eisfeldt and Rampini (2006) try to explain this puzzle through an ad-hoc countercyclical cost to reallocation. Frictions generally considered by the literature such as capital adjustment costs or time-to-build assumptions still satisfy a competitive equilibrium, hence cannot improve on explaining idle capital or procyclical reallocation. Cogley and Nason (1995) additionally show that such aggregate investment constraints do not help generate the output dynamics that we see in the data; positive output growth autocorrelation and hump-shaped impulse response function of output. In this paper we include search frictions to used capital reallocation in order to endogenously justify idle capital and generate procyclical reallocation without an ad-hoc reallocation cost. Additionally, we want to look at the business cycle implications of capital search frictions.

Physical capital is often specific to a task and/or fixed to a particular location. These specificities can potentially result in costly physical capital reallocation. Similar to the labor literature, one can think of such costs as search costs, which vary with the degree of specificity and the business cycle. Depending on the degree of specificity, some part of physical capital remains unmatched hence explaining the existence of idle capital. Unlike the labor market where we observe aggregate unemployment and job vacancy posting rates, there is no such aggregate measure of unused capital.

Figure 1

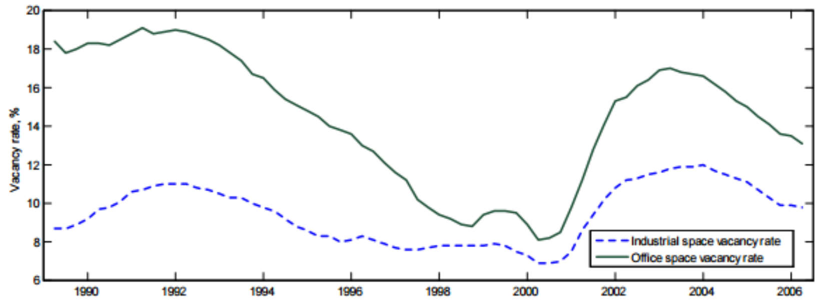

Figure 1: Vacancy rate for multi-tenant industrial and office space; average over 56 metropolitan U.S. markets. Source: Torto Wheaton Richard Ellis.

Kurmann and Petrosky-Nadeau (2007) present some evidence from the market for leased non-residential property, where vacant space is directly observable. Plotting the industrial and office space vacancy rates, they show that vacancies move with the business cycle, being very high during the 1990-1992 recession and gradually decreasing for the rest of the 1990s, increasing again with the 2001 recession. On average, the vacancy rate varies from 9.5% for industrial space to 14.5% for office space.

They also look at the allocation of finished capital goods, which are less specific than non-residential property and find that the allocation rate of such capital goods tends to be closer to unity, with unity indicating no frictions. The countercyclical nature of reallocation in such capital markets and its dependence on specificities makes it easy to compare capital allocation frictions to labor frictions. Therefore, we will follow a similar approach to the labor literature such as Andolfatto (1996) and Den Haan, Ramey and Watson (2000) in modeling these frictions for capital markets.

Our paper is closest to Gavazza (2011), which looks at U.S. commercial aircraft transactions to show that thin markets are less liquid and that the price of assets with thinner markets is lower, resulting in higher price dispersion. In perfectly competitive markets there is no price dispersion, and there should be no delays in between two transactions. Showing that both are non-zero and significantly correlated with market thickness measures, is evidence that even fairly liquid markets like commercial aircrafts are subject to allocation frictions.

In the model that we develop in Section 2, all but capital market is standard as in baseline RBC. We have a continuum of households and firms. Firms purchase either new capital from households or used capital that separates from other firms. New capital equilibrium is Walrasian, while used capital is subject to reallocation frictions that depend on the state of the economy. Thus, the model endogenously results in idle capital, since due to used capital reallocation frictions, part of the used capital that separates will not find a match. The share that separates can be exogenous or endogenously determined by the firm.

The basic intuition why this model can help explain procyclical reallocation is related to the procyclical matching probability (countercyclical congestion). In any period, a share of newly separated capital and an idle capital component from the previous period, both enter the matching market. Defining liquid capital as the sum of these two components, under exogenous separation probability, liquid capital is predetermined. A firm looking for more capital, can purchase new capital from the households or post vacancies for used capital. When a positive productivity shock hits and the demand for capital increases, vacancies will increase while liquid capital stays the same. This will result in a higher probability of liquid capital matching with a vacancy; i.e. procyclical reallocation.

Under endogenous separation, used capital reallocation is not as clear. A positive productivity shock increases the demand for capital, but now liquid capital responds negatively to this shock because separation rate decreases. This results in higher vacancy costs since the probability that a vacancy will match decreases. The firms then will respond by decreasing vacancies. However, due to the increased demand for capital, firms might still want to post more vacancies, up to the point where the total marginal cost of used capital does not exceed total marginal benefit. A decrease in liquid capital with an ambiguous effect on vacancies, makes it hard for us to claim which way used capital reallocation might go.

It is ambiguous whether our model can generate the hump-shaped response of output, but the intuition behind output amplification is as follows. When a positive aggregate shock hits the economy, the demand for capital increases. Due to the available idle capital, firms will adjust their capital immediately. An immediate increase in capital will result in labor increasing by even more, since the marginal product of labor also depends on capital. Immediate amplification of both factors of production will then result in output amplification.

The main contribution of our results is that we observe idle capital, which comes endogenously from the model due to search frictions, the model with exogenous separation can endogenously explain procyclical reallocation and both endogenous and exogenous separation result in amplified response of output. The drawback is that the model with endogenous separation fails to replicate procyclical reallocation, while predicting countercatually procyclical benefits to reallocation.

2 Model

The model environment is as in standard RBC, with the exception of capital markets. There is a continuum of infinitely lived households and a continuum of atomistic firms. Households sell new capital to firms at a unit price ρ_t^n. Following much of the investment literature, we assume that new capital investment is predetermined, since resources need to be diverted from consumption a period in advance. Used capital comes from the existing firms. Firms can invest in either new or used capital. Obtaining used capital is subject to search frictions. When a firm matches with a unit of either new or used capital, a relationship i is formed. The relationship can be successful or it can end in separation. The output production from any surviving relationship i is $Y_{it} = A_t z_{it} {N^*_{z_{it}}}^{1-\alpha}$, where A_t is the aggregate productivity and z_{it} is the relationship i specific idiosyncratic disturbance. $N^*_{z_{it}}$ is chosen optimally by the firm to be

$$N^*_{z_{it}} = \left[\frac{(1-\alpha)A_t z_{it}}{w_t}\right]^{1/\alpha}.$$

In the beginning of every period both aggregate and relationship specific shocks are realized. Then separation might occur or not. The probability that separation occurs for any relationship i is s_{it}. The firm only chooses to separate if the realized idiosyncratic shock is low enough such that it is not optimal to continue producing. More specifically, we define $\underline{z}_{it}$ as the threshold value such that if $z_{it} < \underline{z}_{it}$ the firm chooses to separate and if $z_{it} \geq \underline{z}_{it}$ the firm continues the relationship. Denoting $(1-\delta)K_{t-1}$ the total productive capital after depreciation in period $t-1$, which is the equivalent of total surviving relationships in $t-1$, then in the beginning of period t, a share $s_t(1-\delta)K_{t-1}$ will separate. Due to used capital search frictions, only a fraction of the used capital that separates finds a match, hence additionally there is a total of U_{t-1} units of idle capital that separated but did not find a match in the previous period , which enters period t. An important assumption here is that when a firm and its capital separate, the firm cannot reconnect this unit as productive capital, hence will have to keep it idle. Therefore, the total amount of liquid used capital in the market is $L_t = s_t(1-\delta)K_{t-1} + U_{t-1}$.

The firms that want to purchase this capital, are entrant firms with idiosyncratic shock realization z^{high}. These firms can purchase either new or used capital, satisfying a no-Arbitrage condition $\rho^n_t = \rho^u_t + \kappa\frac{f(\theta)}{\theta}$, where κ is the unit cost of posting a vacancy. This condition makes the entrant indifferent between new and used capital. It says that in order for the firm to have incentive to buy used capital, the price of used capital should be lower than the price of new capital such that it compensates the firm for the search costs of used capital. These firms post an aggregate number of vacancies V_t. Liquid capital is matched with vacancies according to the constant returns to scale matching function $m(V_t, L_t)$. The market thickness will be defined as $\theta_t = \frac{V_t}{L_t}$. Then the probability that a vacancy matches with liquid capital will be $p(\theta_t) = \frac{m(V_t,L_t)}{V_t}$ and the probability that a unit of liquid capital matches with a vacancy is $f(\theta_t) = \frac{m(V_t,L_t)}{L_t}$. From here, $f(\theta_t) = p(\theta_t)\theta_t$. The unit price of used capital is determined through Nash Bargaining, with η being the buyer firm's bargaining power.

A firm does not distinguish between new and used capital in production, therefore the total productive capital will be $K_t = (1 - s_t)(1 - \delta)K_{t-1} + \frac{f(\theta_t)}{\theta_t}V_t + I_t^n$. The rest of the model is standard as in basic RBC. Households draw utility from consumption and leisure $U(C_t, 1-N_t)$, for C_t denoting consumption and N_t being labor supply, and both the labor and the goods market are frictionless.

2.1 Endogenous Separation

Consider any relationship i after a firm matches with a unit of capital in period t. Before the aggregate shock or relationship specific idiosyncratic shock is realized, the relationship might exogenously separate with probability s^x. If this relationship survives exogenous separation, then after the firm observes the idiosyncratic shock realization z_{it}, the firm will decide to separate or not, with endogenous probability of separation s_{it}^n. From here, the total probability of separation for a relationship i in period t will be $s_{it} = s^x + (1 - s^x)s_{it}^n$. Assuming the relationship specific idiosyncratic shock z_{it} follows an iid probability distribution $\mu(z_{it})$, then the endogenous probability of separation will be

$$s^n_{it} = \int_{z_{min}}^{\underline{z}_{it}} d\mu(z_{it}). \tag{1}$$

z_{min} represents the lowest value that the idiosyncratic shock can attain and $\underline{z}_{it}$ is the threshold value, such that a firm with a realization $z_{it} < \underline{z}_{it}$ will choose to separate and a firm with a realization $z_{it} \geq \underline{z}_{it}$ will choose to continue the relationship. This threshold value will be such that at $\underline{z}_{it}$ the firm is indifferent between separating in which case the firm gets the marginal value of idle capital J^u_t and continuing in this relationship and receiving the marginal value of productive capital:

$$A_t \underline{z}_{it} {N^*_{\underline{z}_{it}}}^{1-\alpha} - w_t N^*_{\underline{z}_{it}} + J^k_t = J^u_t. \tag{2}$$

The expected value of future profits for a relationship that survived separation in period t is

$$J^k_t = (1-\delta)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}[(1-s_{t+1})\frac{\int_{\underline{z}_{t+1}}^{\infty}(A_{t+1}z_{t+1}{N^*_{z_{t+1}}}^{1-\alpha} - w_{t+1}N^*_{z_{t+1}} + J^k_{t+1})d\mu(z_{t+1})}{1-M(\underline{z}_{t+1})} + s_{t+1}J^u_{t+1}]. \tag{3}$$

Due to the assumption that the relationship specific shock is independent over time, a firm with a high shock realization in t will not necessarily have a high realization in $t+1$. This implies the expected probability of separation in $t+1$ will be independent of i $s_{it+1} = s_{t+1}$. Similarly, the value of an idle unit of capital entering period t will be; the profits from successfully finding a match in which case the firm gets the price of used capital with probability $f(\theta_t)$ or getting the continuation value with probability $1 - f(\theta_t)$ if it does not find a match

$$J^u_t = f(\theta_t)\rho^u_t + (1 - f(\theta_t))\beta E_t \frac{\lambda_{t+1}}{\lambda_t} J^u_{t+1}. \tag{4}$$

For notation simplicity, from now on we can denote $\beta\frac{\lambda_{t+1}}{\lambda_t} = \Lambda_{t,t+1}$.

The matching of used capital is subject to search frictions, hence there will be some non-zero trade surplus from each unit of used capital traded, which will have to be shared accordingly between the buyer and seller of used capital. Because the shock realization for entering firms is fixed at z^{high}, we do not need to keep track of the distribution of the entrants who are also the buyers of both new and used capital. Therefore, the price of used capital will be determined through Nash Bargaining as the weighted sum of the buyer's and the seller's surplus, with bargaining powers serving as weights

$$\rho_t^u = (1-\eta)[A_t z^{high} {N^*_{z_t^{high}}}^{1-\alpha} - w_t N^*_{z_t^{high}} + J_t^k] + \eta E_t \Lambda_{t,t+1} J_{t+1}^u. \tag{5}$$

The first term in square brackets represents the buyer's surplus from trade, which is the difference between the marginal value of productive capital and the unit price of used capital. If the buyer walks away its outside option would be zero. The second term in square brackets represents the seller's surplus, which is the difference between the unit price of used capital and the marginal value of idle capital. This is because if the seller accepts the bargaining offer, the seller gets the price, but if the seller rejects the offer then it would walk away with one more unit of idle capital.

The entrant will be willing to pay ρ_t^n for a unit of new capital only if the cost of new capital equals the marginal value of productive capital at the shock realization z^{entry} as follows

$$\rho_t^n = A_t z_t^{high} N^{1-\alpha}_{z_t^{high}} - w_t N_{z_t^{high}} + J_t^k \tag{6}$$

Substituting (6) into (5), we can express the used price in terms of the new price

$$\rho_t^u = (1-\eta)\rho_t^n + \eta E_t \Lambda_{t,t+1} J_{t+1}^u \tag{7}$$

2.2 Households

The households part of the model is standard. Households maximize their expected discounted utility over consumption, leisure and investment. The household gets compensated $W_t N_t$ for providing N_t units of labor and $\rho_t^n I_t^n$ for the I_t^n units of new capital sold to the firms. Converting numeraire into new capital will be costly such that there is a per unit convex adjustment cost $F(\frac{I_t}{I_{t-1}})$, where $F(1) = 0, F'(.) > 0, F"(.) > 0$. Then, the households will only sell $I_t^n = (1 - F(\frac{I_{t-1}}{I_{t-2}}))I_{t-1}$. Additionally, the households can use a risk-free asset B_{t+1} to smooth consumption, whose price is normalized to 1 and its return is r_t. Here households also own shares of firms and as such get dividends D_t. Then, the budget constraint of the representative household will be

$$C_t + I_t + B_{t+1} = w_t N_t + \rho_t^n I_t^n + (1 + r_t)B_t + D_t \tag{8}$$

The household then maximizes

$$V(I_{t-1}, I_{t-2}, B_t) = \underset{C_t, N_t, I_t, B_{t+1}}{max} [u(C_t, 1 - N_t) + \beta E_t V(I_t, I_{t-1}, B_{t+1})] \tag{9}$$

subject to the budget constraint.

The first order conditions are standard as in baseline RBC, with the exception of (13) and (14) which also account for the convex adjustment costs.

$$(C_t) : u^t_{C_t} = \lambda_t \tag{10}$$

$$(N_t) : u^t_{N_t} = \lambda_t w_t \tag{11}$$

$$(I_t) : \beta E_t V_{I_t}(I_t, I_{t-1}, B_{t+1}) = \lambda_t \tag{12}$$

$$(B_{t+1}) : \beta E_t V_B(I_t, I_{t-1}, B_{t+1}) = \lambda_t \tag{13}$$

Defining $Q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \rho^n_{t+1}$, we can re-write (13) as follows

$$1 = \beta E_t[Q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} F'(\frac{I_{t+1}}{I_t}) \frac{I^2_{t+1}}{I^2_t}] + Q_t[(1 - F(\frac{I_t}{I_{t-1}})) - F'(\frac{I_t}{I_{t-1}}) \frac{I_t}{I_{t-1}}] \tag{14}$$

If we set $F(.) = 0$ and $F'(.) = 0$ such that there are no adjustment costs, equation (15) collapses to the standard Real Business Cycle model with capital renting equivalent $\lambda_t = \beta E_t \lambda_{t+1} \rho^n_{t+1}$. This shows that the purchasing assumption does not change the implications of the model, while it is a useful assumption to make when we consider used capital markets separately.

2.3 Exogenous Separation

This is a special case of the endogenous separation model, which makes the model more tractable and helps us understand the implications of used capital search frictions without additional assumptions about separation decision. We assume that the relationship specific idiosyncratic shock is the same for all relationships and across time $z_{it} = z, \forall i, t$. It is then safe to drop relationship specific indicator i from the model. Additionally, knowing z with certainty means that now the firm does not endogenously decide whether to separate or not, resulting in a fixed separation probability $s_{it} = s, \forall i, t$. Making the two necessary substitutions $z_{it} = z$ and $s_{it} = s$, $\forall i, t$ we can then solve the exogenous separation case.

3 Calibration

For the calibration of some standard parameters, we follow King and Rebelo (2000). The discount factor β is calibrated such that the annual real interest rate is 6.5%, which means the quarterly interest rate should be $r = \frac{r_{annual}}{400}$. The annual depreciation rate is $d_{annual} = 10\%$, implying a quarterly depreciation $\delta = \frac{d_{annual}}{400}$. We set $\alpha = 0.333$ such that it matches the standard long-run U.S. labor income share $1 - \alpha = 0.667$. To coincide with the U.S. postwar data N is set to 20%. Then specifying utility as $u(C_t, 1 - N_t) = logC_t + \frac{\omega}{1-\xi}(1 - N_t)^{1-\xi}$, we set the parameter that controls the labor supply elasticity $\xi = 1$, which will then imply a value for ω that matches the steady state labor. There is no economic reason to fix $\xi = 1$, however as long as this parameter is the same in both our model and the baseline RBC, this possesses no issue for the comparison. Shutting down adjustment costs such that $F"(.) = 0$, then $Q = 1$ which results in $\rho^n = \frac{1}{\beta}$. Following the same literature, we set the parameter that governs the technology shock persistence to be $\nu = 0.979$ and the standard deviation of technology shock innovation $\sigma = 0.72$. The rest of the calibration is specific to our model. We use the steady state equations in order to calibrate the necessary values in either the exogenous or endogenous specification, which for the majority of cases overlap significantly.

The main challenge here is the calibration of the search related variables. To calibrate the separation rate we use the ratio of total investment to total capital, which is computed by Eisfeldt and Rampini (2006) to the value 0.24 using Compustat data. This data is reported as (price)*(quantity). The equivalent of this ratio in our model is $\frac{\rho^n I^n + \rho^u I^u}{\bar{\rho} K}$. Then, re-writing our total productive capital steady state as $\bar{\rho} K = (1 - s)(1 - \delta)\bar{\rho} K + \rho^n I^n + \rho^u I^u$ we can express

$\rho^n I^n + \rho^u I^u = \bar{\rho} K[1-(1-s)(1-\delta)]$. From here $[1-(1-s)(1-\delta)] = 0.24$, which results in $s = 0.22$. For the exogenous model, this is the exogenous separation rate, whereas for the endogenous model this is the total separation, which in steady state is defined as $s^x + (1-s^x)s^n$.

Using the ratio U/K, which is reported by Kurmann and Petrosky-Nadeau (2007) to be between $9.5\% - 14.5\%$, we can calibrate the probability of liquid capital matching with a vacancy $f(\theta)$, and the following ratios $\frac{L}{K}$ and $\frac{U}{L}$. Re-writing the idle capital steady state equation $\frac{U}{K} = \frac{(1-f(\theta))s(1-\delta)}{f(\theta)}$ and choosing $\frac{U}{K} = 0.1$, we solve for the matching probability to be $f(\theta) = \frac{s(1-\delta)}{s(1-\delta)+\frac{U}{K}} = 0.68$ Then from the liquid capital steady state equation we can express $\frac{L}{K} = s(1-\delta) + \frac{U}{K} = 0.31$, then $\frac{U}{L} = \frac{U/K}{s(1-\delta)+U/K} = 0.32$. Having calibrated $f(\theta)$, we can find J^u from the steady state value function for idle capital to be $J^u = \frac{\beta(1-\eta)f(\theta)\rho^n}{1+\beta(1-\eta)(1+f(\theta))}$. To do so, we need to specify a value for the buyer's bargaining power η. Having no additional information related to this parameter for now we set $\eta = 0.5$, which assigns equal power to both the buyer and the seller. Then from the bargaining price steady state equation we can calibrate $\rho^u = (1-\eta)\rho^n + \eta j^u$. Knowing the used capital steady state value, we can further calibrate $\kappa\frac{\theta}{f(\theta)}$ from the no-Arbitrage condition to be $\kappa\frac{\theta}{f(\theta)} = \rho^n - \rho^u$.

To calibrate market thickness θ, we adopt the same CRTS matching function as Den Haan, Ramey and Watson (2000); $m(V_t, L_t) = \frac{V_t L_t}{(V_t^\epsilon + L_t^\epsilon)^{1/\epsilon}}$. This specification has the appealing feature that it results into matching probabilities within the unit interval. Then, having calibrated $f(\theta)$ and setting $\epsilon = 0.5$ we can find the market thickness from $f(\theta) = (\frac{\theta^\epsilon}{\theta^\epsilon+1})^{1/\epsilon}$ to be $\theta = \frac{f(\theta)}{(1-f(\theta)^\epsilon)^{1/\epsilon}}$. For the exogenous model the rest of the necessary steady states follow from here.

The endogenous separation case has some additional parameters that we need to consider. First we specify the relationship specific idiosyncratic shock to follow a Pareto Distribution with shape parameter ϕ and scale parameter z_{min} such that $\mu(z) = \frac{\phi z_{min}^\phi}{z^{\phi+1}}$. Then in order to calibrate the rest of the endogenous separation model, we need to specify what z_{min} and ϕ are. Without loss of generality, we set $z_{min} = 1$. In order to get a closed form solution for output in the endogenous model, we additionally need to have ϕ such that $E(z^{1/\alpha})$ exists. By the properties of the Pareto Distribution, $\phi > \frac{1}{\alpha}$ for the closed form solution to exist. This implies that $\phi > 3$, therefore in this phase of the project we will arbitrarily set $\phi = 5$. Then setting the exogenous separation $s^x = 0$ such that we attribute all separation to the firm's endogenous decision, results in $s^n = s$ and from here we can calibrate $\underline{z}$ to be $\underline{z} = z_{min}(1 - s^n)^{-1/\phi}$, using the definition of the endogenous separation. The rest follows from here. Combining the productive capital value function with the idiosyncratic shock threshold equation we can find both J^k and w to be $J^k = \frac{\beta(1-\delta)[(1-s)(\frac{\phi}{\phi-1/\alpha})(f(\theta)\rho^u+(1-f(\theta))J^u)+sJ^u]}{1+\beta(1-\delta)(1-s)\frac{1/\alpha}{\phi-1/\alpha}}$ and $w = (\frac{[(1-\alpha)A\underline{z}]^{1/\alpha}\frac{\alpha}{1-\alpha}}{f(\theta)\rho^u+(1-f(\theta))J^u-J^k})^{\frac{\alpha}{1-\alpha}}$. The last parameter value necessary for the endogenous steady state system to be fully calibrated is z^{entry}. After finding w and J^k, z^{entry} follows from the new capital price equation to be $z^{entry} = \frac{[(\rho^n - j^k)(\frac{1-\alpha}{\alpha})]^\alpha w^{1-\alpha}}{A(1-\alpha)}$.

4 Results

In this section we compare our search model to the standard RBC model. The aggregate productivity shock is specified to be the same in all three cases discussed below, which makes the impulse responses directly comparable. The search model with and without endogenous separation shows some significant amplification as compared to the standard RBC. The output amplification comes from the amplification in both capital and labor. In standard RBC capital is predetermined, whereas in the search model capital jumps up immediately. Since marginal productivity of labor depends on capital, labor responds even more resulting in significant output increase. The model also does a very good job at endogenously generating idle capital u, which under both exogenous and endogenous separation is countercyclical.

When we set the separation rate to be exogenous, the search model additionally results in

Figure 1

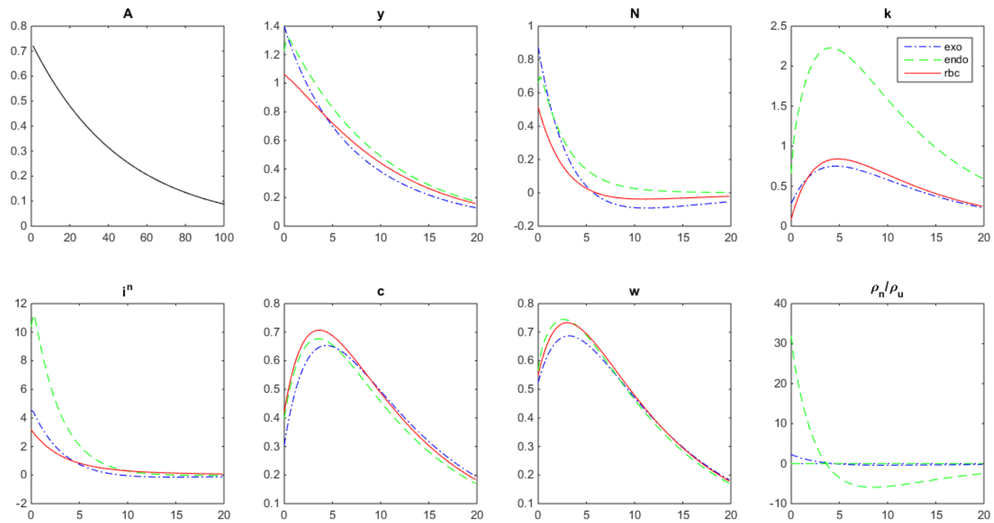

Figure 1: This graph compares our search model with and without endogenous separation, to the baseline RBC. The shock is the same in all three specifications.

Figure 2

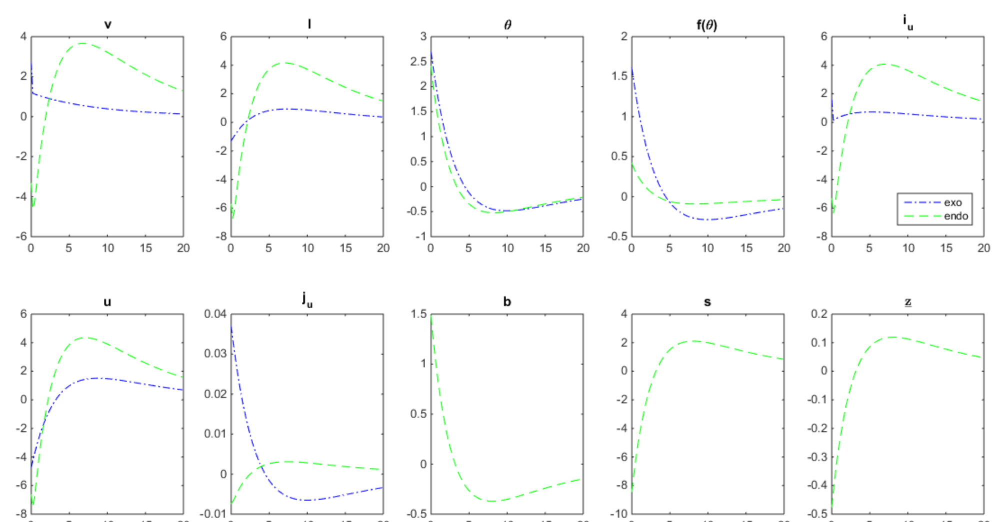

Figure 2: This graph consists of the variables specific to the search model only.

procyclical used capital reallocation i_u, which is what we see in the data. Liquid capital is fixed,[1] but vacancies go up as a response to an increase in a since firms demand for capital increases. θ by definition increases unambiguously, because l if fixed and v goes up, and so does the probability that liquid capital matches with a vacancy $f(\theta)$. This explains why we see procyclical separation. To get from the endogenous to the exogenous separation, we had to set $z_{it} = z, \forall i, t$ and $s_{it} = s, \forall i, t$, therefore separation rate, separation threshold and reallocation benefits variables do not appear for the exogenous separation case. They are all zero under the log-linear specification.

To understand the discrepancies between the search model with exogenous separation and the same model with endogenous separation, we look at the impulse responses of the key search variables in Figure 2. The model with endogenous separation looses the used capital reallocation procyclicality that we observe in the data. This is because a positive aggregate shock decreases the separation rate, hence by definition liquid capital reduces by a lot. All else equal, the probability of a vacancy finding a match goes down, hence the entrant firm's cost of searching for used capital increases, which additionally explains why vacancies go down. A higher cost of searching for used capital, implies a lower incentive to purchase used capital. The entrants then will purchase more new capital, which explains the large jump in new investment in Figure 1.

Benefits to reallocation b also appear counterfactually procyclical. The force driving the benefits to reallocation here is the separation threshold $\underline{z}$. The benefit to reallocation here is defined similar to Eisfeldt and Rampini (2006), as the productivity spread. In our model, we have made the restricting assumption that the productivity of the buyer firms is exogenously fixed to some z^{entry}, hence the only variable affecting this spread is $\underline{z}$. Since $\underline{z}$ appears to be highly countercyclical, then the spread will be higher during upturns and lower during downturns, hence procyclical.

[1]Liquid capital l is specified as predetermined, and as such in the graph the liquid capital response that we see in period 0 corresponds to its response one period in the future.

The high negative response of the separation threshold $\underline{z}$ to a positive aggregate shock is due to the increase in current and future expected aggregate productivity. When a positive shock hits the economy all else equal, total productivity increases. This results in higher profits from not separating, hence the threshold goes down. A decrease in the separation threshold, means a lower share of firms will be in the separation region, in other words separation will decrease. This additionally results in a negative jump in liquid capital, and as a result a decrease in total idle capital u.

The high initial jump in total capital comes from two components; the part of relationships that survive separation and the new capital purchases. A lower separation results into a higher share of surviving relationships which increases total capital. Additionally, an increase in new capital purchases increase total capital. In the exogenous separation, the first component was predetermined that is why the initial response of total capital for the exogenous case is smaller. The big negative jump in used capital investment for the endogenous case should affect total capital negatively, however the positive immediate jump of the later shows that the first two components' effect dominates.

5 Conclusion

In this paper, we incorporated used capital search frictions to an otherwise standard Real Business Cycle model. The three stylized facts that we wanted to replicate using our search model are; idle capital exists in equilibrium, output response should be amplified and hump shaped and used capital reallocation should be procyclical with countercyclical benefits to reallocation. The search model performs very well at generating countercyclical idle capital in equilibrium. We additionally get significant output amplification with some slight hump shape response of output in the endogenous model. The exogenous separation case manages to replicate procyclical capital reallocation, without the need of ad-hoc reallocation costs. The results related to used capital reallocation appear counterfactual under endogenous separation. Opposite to the data facts, the model with endogenous separation results in procyclical benefits to reallocation and countercyclical used capital reallocation. One possible way to tackle this problem would be to allow the entrants productivity to also be endogenously determined by the model. Entrant's productivity will, by the definition of benefits to reallocation, affect the productivity spread and as a result the used capital reallocation. The next step from here is running robustness checks with respect to the bargaining power and allowing for capital adjustment costs to be non-zero. Other possible avenues would be considering other matching functions, such as Increasing or Decreasing Returns to Scale functions, to see whether our results are robust or they depend on the properties of the matching function.

References

[1] Andolfatto, David (1996). *"Business Cycles and Labor-Market Search"*. The American Economic Review, Vol. 86, No. 1, pp. 112-132.

[2] Boldrin, Christiano and Fisher (2001). *"Habit Persistence, Asset Returns, and the Business Cycle,"*. American Economic Review, vol. 91(1), pp. 149- 166.

[3] Cogley and Nason (1995). *"Output Dynamics in Real-Business-Cycle Models"*. The American Economic Review, Volume 85, Issue 3, 492-511.

[4] Den Haan, Ramey and Watson (2000). *"Job Destruction and Propagation of Shocks"*. The American Economic Review, vol. 90(3), pp.482-498.

[5] Eisfeldt and Rampini (2006). *"Capital Reallocation and Liquidity"*. Journal of Monetary Economics, Volume 53(3), pp. 369-399.

[6] Gavazza, Alessandro (2011). *"The Role of Trading Frictions in Real Asset Markets."*. American Economic Review, Volume 101(4), pp. 1106-1143.

[7] King and Rebello (2000) *"Resusciating Real Business Cycles"*. National Bureau of Economic Research. Working Paper 7534.

[8] Ramey and Shapiro (1998). *"Costly capital reallocation and the effects of government spending"*. Working paper.